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Elemental solution of the ultrahyperbolic equation iterated m times

Solución elemental de la ecuación ultrahiperbólica iterada m veces

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ABSTRACT

Let P(x) be a quadratic form defined by (1). We know from (Gelfand and Shilov 1964)) that the equation

$$L_P\{u(x_1, x_2, .. x_n)\} = f(x_1, ... x_n)$$

is often called ultrahyperbolic, where P=P(x) is defined by (31) and L_P by (5). In this article we give a sense the elementary solution of the equation

$$L_P^m\{u(x_1, x_2, .. x_n)\} = f(x_1, ... x_n)$$

where L_P^m is the operator L{P} iterated m- times.$L_P^m\{u(x_1, x_2, .. x_n)\} = f(x_1, ... x_n)$

Keywords: Distributions, equations, solution elementary, ultrahyperbolic
Classification MSC.: 46F10, 46F12.

RESUMEN

Sea P(x) una forma cuadrática definida por (1). Sabemos de (Gelfand and Shilov 1964)) que la ecuación

$$L_P\{u(x_1, x_2, .. x_n)\} = f(x_1, ... x_n)$$

es con frecuencia llamada ultrahiperbólica, donde P=P(x) es definida por by (31) y L_P por (5). En este artículo nosotros damos un sentido a la solución elemental de la ecuación

$$L_P^m\{u(x_1, x_2, .. x_n)\} = f(x_1, ... x_n)$$

donde L_P^m es el operador L_P iterado m veces.

Palabras claves: Distribuciones, ecuaciones, solución elemental, ultrahiperbólico
Clasificación según MSC.: 46F10, 46F12.

1. INTRODUCCIÓN

Let $(x_1, x_2, .. x_n)$ be a point in the n-dimensional Euclidean space and let P be nondegerate quadratic form defined by

$$P((x_1, x_2, .. x_n)) = x_1^2 + \cdots + x_\mu^2 - x_{\mu+1}^2 - \cdots - x_{\mu+v}^2 \quad (1)$$

where $\mu + v$ =n is the dimension of the spacer. The P=0 hypersurface is a hypercone with a singular point(the vertex) at the origin.
We defined the generalized function$< P_+^\lambda, \varphi >$ whereλ is a complex number, by the following form

$< P_+^\lambda, \varphi > = \int P^\lambda \varphi(x) dx$, para P>0 (2)

(Gelfand and Shilov, 1964), where $x = dx_1 .. dx_n$,$\varphi \in C_0^\infty$is the space of infinitely differentiable function with compact support. For Re(λ)≥0,the integral defined in(2) converges and are analytic function of λ.

Analytic continuation to Re(λ)<0 can be used to extend the definition $< P_+^\lambda, \varphi >$.

We know from(Gelfand and Shilov,1964) that $P_+^\lambda\}$ have two sets of singularities, namely λ=-k,k=1,2,3,... and λ=-(n/2) -s,s=0,1,2,... and the following formulae are valid

$$Res P_{+\,\lambda=-k}^\lambda = (-1)^{k-1} \frac{1}{(k-1)!} \delta^{(k-1)}(P) \text{ if n is odd (μ odd and ν even),} \quad (3)$$

$$Res P_{+\,\lambda=-k}^\lambda = (-1)^{k-1} \frac{1}{(k-1)!} \delta^{(k-1)}(P) \text{ if n is even but k<(n/2),} \quad (4)$$

and

$$Res P_{+\,\lambda=-\frac{n}{2}-s}^\lambda = (-1)^{\frac{v}{2}} \pi^{\frac{n}{2}} = \frac{L^s\{\delta\}}{s! 4^s \Gamma(\frac{n}{2}+s)} \text{ if n is odd (μ odd and ν even)} \quad (5)$$

$$Res P_{+\,\lambda=-k}^\lambda = (-1)^{k-1} \frac{1}{(k-1)!} \delta^{(k-1)}(P) \text{ if μ and ν are both even (n even) and k<(n/2),} \quad (6)$$

$$Res P_{+\,\lambda=-\frac{n}{2}-s}^\lambda = (-1)^{\frac{v}{2}} \pi^{\frac{n}{2}} = \frac{L^s\{\delta\}}{s! 4^s \Gamma(\frac{n}{2}+s)} \quad (7)$$

if μ and ν are both even and s≥(n/2) (Gelfand and Shilov,1964, page353) and (Aguirre,2010)

$$Res P_{+\,\lambda=-k}^\lambda = (-1)^{k-1} \frac{1}{(k-1)!} \delta^{(k-1)}(P) \quad (8)$$

if μ and ν are both odd (n even) and k<(n/2)

$$Res P_{+\,\lambda=-\frac{n}{2}-s}^\lambda = (-1)(-1)^{\frac{v-1}{2}} \pi^{\frac{n-1}{2}} [\psi((\mu/2)) - \psi((n/2))] \frac{L^s\{\delta\}}{s! 4^s \Gamma(\frac{n}{2}+s)} \quad (9)$$

if μ and ν are both odd and s≥(n/2)

where

$$\psi(h) = -\gamma + 1 + (1/2)+\ldots+(1/(h-1)), h = 2,3,\ldots \tag{10}$$

$$\psi(h + (1/2)) = -\gamma - 2\ln 2 + 2(1 + (1/3)+\ldots+(1/(2h-1)), h = 1,2,3,\ldots) \tag{11}$$

γ is the Euler`s constant and

$$L = \sum_{i=1}^{\mu} \frac{\partial^2}{\partial x_i^2} - \sum_{i=\mu+1}^{\mu+\upsilon} \frac{\partial^2}{\partial x_i^2} \tag{12}$$

On the other hand, from(Aguirre,2015)) the Fourier transform of P_+^{λ} is given by the following formula

$$F\{P_+^{\lambda}\}=\pi^{\frac{n}{2}}\, 2^{2\lambda+n} \frac{\Gamma(\lambda+1)\Gamma(\lambda+(n/2))}{(\Gamma(1+\lambda+(\nu/2))\Gamma(-\lambda-(\nu/2))))} (Q(y))^{-\lambda-(n/2)} \tag{13}$$

if $Q(y)$>0, where $F\{f(x)\} = \int_{R^n} e^{-i<x,s>} f(x)dx,$

(14)

$$<x,s>=x_1 s_1 +,\ldots x_n s_n \tag{15}$$

and

$$Q(y_1, y_2,\ldots y_n)) = y_1^2 + \cdots + y_\mu^2 - y_{\mu+1}^2 - \cdots - y_{\mu+\upsilon}^2. \tag{16}$$

Now, from(Gelfand and Shilov,1964),page258-259 we have the following formula

$$L^m\{P^{\lambda+m}\} =$$

$$= 2^{2m}(\lambda + 1)\ldots(\lambda + m)(\lambda + (n/2))((\lambda + (n/2) + 1)\ldots(\lambda + (n/2) + m - 1)P^{\lambda} \tag{17}$$

2. GENERAL CONSIDERATION

In this article we obtain elementary solution of the equatiuon $L_P^m\{u(x_1, x_2,\ldots x_n)\} = f(x_1, x_2,\ldots x_n)$ taking into accounbt the dimension of the space. First considering the cases n odd and the condition m<n/ 2 and the the case m>n /2. Finally we are going to study the case n even f $m \geq (n/2)$or $m \geq \left(\frac{n}{2}\right)$, $m < n/2$ and considering μ and υ both even and μ and υ both μ and υ both odd.

3. ELEMENTARY SOLUCIONS OF THE EQUATION

$$L_P^m\{u\} = f(x_1,\ldots x_n)$$

Let L_P be the operator defined by(11), we are going to study elementary solutions of the equation of the form

$$L_P^m\{\mathrm{u}(x_1, x_2, .. x_n)\} = \mathrm{f}(x_1, x_2, .. x_n) \quad (18)$$

We know that E is elementary solution of (18) if

$$L_P^m\{\mathrm{E}(x_1, x_2, .. x_n)\} = \delta(\mathrm{x}) = \delta(x_1, x_2, .. x_n)\}. \quad (19)$$

From(18), we have

$$\mathrm{F}\{L_P^m\{E\}\} = \mathrm{F}\{\ \delta(\mathrm{x})\} = 1, \quad (20)$$

therefore

$$\mathrm{F}\{E\}\} = (-1)^m (Q(y)^{-m}\}. \quad (21)$$

Now we are going to obtain the elementary solutions E of the equation (18).From(8), we have

$$(Q(y))^{-\lambda-\frac{n}{2}} = \frac{\Gamma(1+\lambda+(\nu/2))\Gamma(-\lambda-(\nu/2)}{\pi^{\frac{n}{2}} 2^{2\lambda+n} \Gamma(1+\lambda)\Gamma(\lambda+(n/2)} \mathrm{F}\{P_+^\lambda\} \quad (22)$$

Now using the definition of inverse of the Fourier transform

$$\mathrm{F}^{-1}\{\mathrm{f(x)}\} = \int_{R^n} e^{-i<x,s>} \mathrm{F}\{\mathrm{f(x)}\}(\mathrm{s}) ds \quad (23)$$

From(21), we have

$$E = \mathrm{F}^{-1}\{-1)^{-m} (Q(y))^{-m} \quad (24)$$

Now we are going to study (22) considering that P_+^λ and $((Q(y))^\lambda\}$ have poles at $\lambda = -(n/2) - j, j = 0,1,2, \ldots$ and $\lambda = -k, k = 1,2,3, \ldots$

2.1:Case1: n odd

2.1.1.: $m < (\frac{n}{2})$. In this case m≠-(n/2)-j, j=0,1,2,... , P_+^λ is regular at λ=m-(n/2), but $((Q(y))^{-\lambda-(n/2)}$ at λ=m-(n/2), has simple poles.

Therefore the values of the function $((Q(y))^\alpha$ at α =-m which we shall call finite part of $((Q(y))^\alpha\}$ at α =-m correspond to regular part of the Laurent expansion of $((Q(y))^\alpha$ about α =-m, namely

$$Q^m \mid_{\alpha=-m} = Q^{-m} = \mathrm{P.f}\{((Q(y))^\alpha\}_{\alpha=-m} = \lim_{\alpha \to -m} (\frac{\partial}{\partial \alpha}(\alpha + m)\ ((Q(y))^\alpha \text{ for } m<(n/2), \quad (25)$$

therefore, from (22)and (24) we have,

$$E = \mathrm{F}^{-1}\{-1)^{-m} ((Q(y))^{-m_1}\} = \frac{(-1)^m \Gamma((n/2)-m)}{2^{2m} \pi^{\frac{n}{2}} \Gamma(m)(-1)^{\frac{v}{2}}} P_+^{m-\frac{n}{2}}. \quad (26)$$

It´s clear that E defined by (26) verify the property (19). In fact, using the property (17) and the formula.

$\Gamma(\lambda + m + 1)/\Gamma(\lambda + 1))$ =(λ+1)...(λ+m) ((A.Erdelyi,1953), pages3-4) . (27)

we have

$$L_P^m\{P_+^{\lambda+m}\} =$$

$$= 2^{2m}(\lambda+1)\ldots(\lambda+\mathrm{m})(\lambda+(\mathrm{n}/2))((\lambda+(\mathrm{n}/2)+1)\ldots(\lambda+(\mathrm{n}/2)+\mathrm{m}-1)P_+^\lambda$$

$$= 2^{2m}\frac{((\Gamma(\lambda+\mathrm{m}+1))}{(\Gamma(\lambda+1)))}.(\left(\lambda+\left(\frac{\mathrm{n}}{2}\right)+1\right)\ldots\left(\lambda+\left(\frac{\mathrm{n}}{2}\right)+\mathrm{m}-1\right)[\left(\lambda+\left(\frac{\mathrm{n}}{2}\right)\right)P_+^\lambda]. \quad (28)$$

By putting λ=-(n/2) in (28) and using the formula (3) we have

$$\lim_{\lambda\to-\frac{n}{2}} 2^{2m}\frac{((\Gamma(\lambda+\mathrm{m}+1))}{(\Gamma(\lambda+1)))}.[(1.2\ldots m-1)].$$

$$.\lim_{\lambda\to-\frac{n}{2}}[\left(\lambda+\left(\frac{\mathrm{n}}{2}\right)\right)P_+^\lambda] \quad (29)$$

From (26), (29) and using the formula $\Gamma(\mathrm{z})\Gamma(1-\mathrm{z}) = (\pi/(\sin(\pi\mathrm{z})))$ (A. Erdelyi, pages 3 − 4), we obtain

$$L_P^m\{\mathrm{E}\}= \delta(x). \quad (30)$$

Therefore E define by (26) for the case n odd is elementary solution of the equation (18).

2.1.2.: n odd, $m >(\frac{n}{2})$. In this case $(Q(y))^{-\lambda-(\mathrm{n}/2)}$ at λ=m-(n/2), has simple poles. Therefore the values of the function Q^γ at γ=-m we means finite part of Q^γ at γ=-m, and correspond to regular part of the Laurent expansion of Q^γ about γ=-m, namely

$$Q^\gamma=\ldots\frac{\mathrm{A}_{-1}}{(\gamma+\mathrm{m})}+\mathrm{A}_0+\sum_{j\geq 0}A_j(\gamma+\mathrm{m})^j \quad (31)$$

Where

$$\mathrm{A}_0=Q^\gamma\,|_{\gamma=-\mathrm{m}}=Q^{-\mathrm{m}}=\mathrm{P.f}\{Q^\gamma\}_{\gamma=-\mathrm{m}}\}=$$

$$= \lim_{\gamma\to-\mathrm{m}}\partial/(\partial\gamma))[(\gamma+\mathrm{m})Q^\gamma] \text{ for m>(n/2)} \quad (32)$$

It should be emphasized that the generalized function $\{Q^{-\mathrm{m}}\}$ is not the value of $\{Q^\gamma\}$ at γ=-m, $Q^{-\mathrm{m}}$ has a simple poles and therefore does not exist at this point. Nevertheless the function $\{Q^{-\mathrm{m}}\}\}$ is in a certain sense a regularization of the ordinary function $Q^{-\mathrm{m}}$.

Similarly from (25) and (26), we obtain the following formula

$$\mathrm{E} = \mathrm{F}^{-1}\{(-1)^{-m}Q^{-\mathrm{m}}\} = (-1)^{-m}\frac{\Gamma((\mathrm{n}/2)-\mathrm{m}))}{2^{2m}\pi^{\frac{n}{2}}\Gamma(m)(-1)^{\frac{v}{2}}}P_+^{m-\frac{n}{2}} \text{ if n is odd and m>(n/2),} \quad (33)$$

where

$$\mathrm{F}\{P_+^\lambda\} = \pi^{\frac{n}{2}}\,2^{2\lambda+\mathrm{n}}\frac{\Gamma(\lambda+(\mathrm{n}/2))(-1)^{\frac{v}{2}}}{\Gamma(-\lambda))}(Q(y)^{-\lambda-(\mathrm{n}/2)} \text{if } \nu \text{ is even and } \mu \text{ odd} \quad (34)$$

and

$$F\{P_+^\lambda\} = \pi^{\frac{n}{2}}\, 2^{2\lambda+n}\, \frac{\Gamma(\lambda+(n/2))\Gamma(\lambda+1)(-1)^{\frac{v-1}{2}}}{\pi} \cos(-\lambda)\pi.(-1)(Q(y))^{-\lambda-(n/2)}$$

if ν is odd and μ even; ν+μ=n dimension of the space. (35)

2.2 :Case 2: n even

2.2.1.: n even, μ and ν are both even and $m < (n/2)$.In this case m=1,2,3,...(n/2)-1, $Q^{-(n/2)-\lambda}$ and $\{P_+^\lambda\}$ have simple poles at λ=m-(n/2) therefore Q^{-m} is finite part of $Q^\gamma\}$ at γ=-m, and $P_+^{-((n/2)-m)}$ is finite part of $\{P_+^\lambda\}$ at λ=-((n/2)-m). From (24), using (34) and (35), we have

$$(Q(y)^{-\lambda-(n/2)}\ |_{\lambda=m-(n/2)} = \mathrm{P.f}\,\{Q^\gamma\}_{\gamma=-m} = \lim_{\gamma\to-m} \partial/(\partial\gamma))[(\gamma+m)Q^\gamma] =$$

$$= \lim_{\gamma\to-m} \partial/(\partial\gamma))[(\gamma+m)C_{\gamma,n}F\{P_+^{-\gamma-\frac{n}{2}}\}] \qquad (36)$$

where

$$C_{\gamma,n}n\} = \frac{\Gamma(\gamma+(n/2))}{(-1)^{\frac{v}{2}}\pi^{\frac{n}{2}}2^{-2\gamma\Gamma(-\lambda)}} \qquad (37)$$

From (36) and (37), we have

$$(Q(y)^{-m} = \lim_{\gamma\to-m} C_{\gamma,n}\,\partial/(\partial\gamma))[(\gamma+m)F\{P_+^{-\gamma-\frac{n}{2}}\}] +$$

$$+\lim_{\gamma\to-m}[(\gamma+m)F\{P_+^{-\gamma-\frac{n}{2}}\}\,\partial/(\partial\gamma))C_{\gamma,n}] \qquad (38)$$

Now taking into account that

$$\lim_{\gamma\to-m} \partial/(\partial\gamma))[(\gamma+m)\{P_+^{-\gamma-\frac{n}{2}}\}] =$$

$$\lim_{\lambda\to-(\frac{n}{2}-m)} \partial/(\partial\lambda))[(\lambda+(\frac{n}{2}-m))\{P_+^\lambda\}] = \mathrm{P.f}\,\{P_+^\lambda\}_{\lambda=-(\frac{n}{2}-m)} \qquad (39)$$

and

$$\lim_{\gamma\to-m}(\gamma+m)\,P_+^{-\gamma-\frac{n}{2}} = \lim_{\lambda\to-(\frac{n}{2}-m)}(\lambda+(\frac{n}{2}-m))\,P_+^\lambda =$$

$$= \frac{(-1)(-1)^{\frac{n}{2}-m-1}}{\left(\frac{n}{2}-m-1\right)!}\,\delta^{\left(\frac{n}{2}-m-1\right)}(P). \qquad (40)$$

we have

$$(Q(y)^{-\mathrm{m}} = \lim_{\gamma\to-\mathrm{m}} C_{\gamma,\mathrm{n}}\, \partial/(\partial\gamma))[(\gamma+\mathrm{m})\mathrm{F}\{P_+^{-\gamma-\frac{n}{2}}\}] +$$

$$+\lim_{\gamma\to-\mathrm{m}}[(\gamma+\mathrm{m})\mathrm{F}\{P_+^{-\gamma-\frac{n}{2}}\}\, \partial/(\partial\gamma))C_{\gamma,\mathrm{n}}] .$$

$$.\left[\lim_{\gamma\to-\mathrm{m}}[\frac{2^{2\gamma}2ln2\Gamma\left(\gamma+\frac{n}{2}\right)+2^{2\gamma}\frac{d}{d\gamma}\Gamma\left(\gamma+\frac{n}{2}\right)}{\Gamma(-\gamma)}] + \lim_{\gamma\to-\mathrm{m}}[\frac{2^{2\gamma}\Gamma\left(\gamma+\frac{n}{2}\right)\frac{d}{d\gamma}\Gamma(-\gamma)}{(\Gamma(-\gamma))^2}]\right]=$$

$$= \left[\frac{2^{-2\mathrm{m}}2ln2\Gamma\left(-\mathrm{m}+\frac{n}{2}\right)+}{\Gamma(-\mathrm{m}+\frac{n}{2})}\right].\,\mathrm{F}\left\{\mathrm{p.f.}\,P_+^{-\left(\frac{n}{2}-m\right)}\right\} +$$

$$+\pi^{-\frac{n}{2}}(-1)^{-\frac{v}{2}}\mathrm{F}\left\{\frac{(-1)(-1)^{\frac{n}{2}-m-1}}{\left(\frac{n}{2}-m-1\right)!}\delta^{\left(\frac{n}{2}-m-1\right)}(P)\right\}.\left[[\frac{2^{-2\mathrm{m}}2ln2\Gamma\left(-\mathrm{m}+\frac{n}{2}\right)+2^{2\gamma}\Gamma'(\frac{n}{2}-m)}{\Gamma(m)}] + [\frac{2^{-2m}\Gamma\left(\frac{n}{2}-m\right)\Gamma'(m)}{(\Gamma(m))^2}]\right]$$

(41)

From (41) have,

$$\mathrm{F}^{-1}\{(-1)^m Q^{-\mathrm{m}}\} = (-1)^m \frac{\Gamma((\mathrm{n}/2)-\mathrm{m}))}{2^{2m}\pi^{\frac{n}{2}}\Gamma(m)(-1)^{\frac{v}{2}}}\{p.fP_+^{-\left(\frac{n}{2}-m\right)}\} +$$

$$+\frac{(-1)^m 2^{-2m}\pi^{-\frac{n}{2}}(-1)^{\frac{n}{2}}\ (-1)^{-\frac{v}{2}}}{\Gamma(m)}[2\ln2+\psi((\mathrm{n}/2)-\mathrm{m})+\psi(\mathrm{m})]\ \delta^{\left(\frac{n}{2}-m-1\right)}(P)$$

if n is even, μ and ν are both even, (42)

where

$$\psi(\mathrm{z}) = \frac{\Gamma'(z)}{(\Gamma(\mathrm{z}))}$$ ((Erdelyi A. 1953,pages 3-4)). (43)

From (24) and considering he formula (31) we obtain that

$$\mathrm{E}=\mathrm{F}^{-1}\{(-1)^{-m}Q(y)^{-m}\}=(-1)^m \frac{\Gamma((\mathrm{n}/2)-\mathrm{m}))}{2^{2m}\pi^{\frac{n}{2}}\Gamma(m)(-1)^{\frac{v}{2}}}\{p.fP_+^{-\left(\frac{n}{2}-m\right)}\} +$$

$$+\frac{(-1)^m 2^{-2m}\pi^{-\frac{n}{2}}(-1)^{\frac{n}{2}}\ (-1)^{-\frac{v}{2}}}{\Gamma(m)}[2\ln2+\psi((\mathrm{n}/2)-\mathrm{m})+\psi(\mathrm{m})]\ \delta^{\left(\frac{n}{2}-m-1\right)}(P) \qquad (44)$$

is elementary solution of the equation (18) for n even and m<(n/2).

2.2.2.: n even, μ and ν are even odd and $m \geq (n/2)$.In this case $(Q(y))^{-\lambda-(n/2)}$ has simple poles at λ=m-(n/2)and P_+^{λ} is regular a λ=m-(n/2), therefore $Q(y)^{-m}$ is finite part of $(Q(y))^{\gamma}$ at γ=-m.

$$Q^{\lambda-\frac{n}{2}}\,|_{\lambda=m-\frac{n}{2}} = (Q(y)^{-m} = \mathrm{P.f}\,\{Q^{\gamma}\}_{\gamma=-m} =$$

$$= \lim_{\gamma\to -m} \partial/\partial\gamma[\frac{(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)}{(-1)^{\frac{v}{2}}\,\pi^{\frac{n}{2}}\,2^{-2\gamma\Gamma(-\lambda)}}\,\mathrm{F}\{P_+^{-\gamma-\frac{n}{2}}\}] =$$

$$= \lim_{\gamma\to -m}\{\frac{(-1)^{\frac{v}{2}}\,\pi^{\frac{n}{2}}\,2^{-2\gamma\Gamma(-\lambda)}.\partial/\partial\gamma[(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)\mathrm{F}\left\{P_+^{-\gamma-\frac{n}{2}}\right\} - [(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)\mathrm{F}\left\{P_+^{-\gamma-\frac{n}{2}}\right\}.\partial/\partial\gamma(-1)^{\frac{v}{2}}\,\pi^{\frac{n}{2}}\,2^{-2\gamma}\Gamma(-\lambda)}{[(-1)^{\frac{v}{2}}\,\pi^{\frac{n}{2}}\,2^{-2\gamma\Gamma(-\lambda)}]^2}\}$$

$$= \lim_{\lambda\to -m}\frac{1}{\pi^{\frac{n}{2}}2^{-2\gamma\Gamma(-\gamma)(-1)^{\frac{v}{2}}}}\lim_{\lambda\to -m}\partial/\partial\gamma[(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)\mathrm{F}\left\{P_+^{-\gamma-\frac{n}{2}}\right\} +$$

$$+\lim_{\gamma\to -m}(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)\mathrm{F}\left\{P_+^{-\gamma-\frac{n}{2}}\right\}.\ln P_+.(-1) - .$$

$$-\lim_{\gamma\to -m}(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right)\mathrm{F}\left\{P_+^{-\gamma-\frac{n}{2}}\right\}.[\frac{\pi^{\frac{n}{2}}2^{-2\gamma\Gamma(-\gamma)(-1)^{\frac{v}{2}}}+\pi^{\frac{n}{2}}2^{-2\gamma\Gamma'(-\gamma)(-1)(-1)^{\frac{v}{2}}}}{(\pi^{\frac{n}{2}}2^{-2\gamma\Gamma(-\gamma)(-1)^{\frac{v}{2}}})^2} \qquad (45)$$

Now, using the following properties

$$\lim_{\gamma\to -m}(\gamma+m)\Gamma(\gamma+(\frac{n}{2}))=(\gamma+m)\,\gamma(\gamma+1)\ldots(\gamma+\frac{n}{2}-1)\Gamma(\gamma) =$$

$$((-1)^{\frac{n}{2}}(-\gamma)(-\gamma-1)\ldots(-\gamma-\frac{n}{2}+1)(\gamma+m)\Gamma(\gamma) =$$

$$= \lim_{\gamma\to -m}\frac{(-1)^{\frac{n}{2}}\Gamma(1-\gamma)}{\Gamma(-\gamma-\frac{n}{2}+1)}(\gamma+m)\Gamma(\gamma) =$$

$$= \frac{(-1)^{\frac{n}{2}}\Gamma(1+m)}{\Gamma\left(m-\frac{n}{2}+1\right)}res_{\gamma=-m}\Gamma(\gamma) = \frac{(-1)^{\frac{n}{2}}\Gamma(1+m)}{\Gamma\left(m-\frac{n}{2}+1\right)}.\frac{(-1)^m}{m!} = \frac{(-1)^{\frac{n}{2}}(-1)^m}{\Gamma\left(m-\frac{n}{2}+1\right)} \qquad (46)$$

for n even and m≥(n/2) and

$$\lim_{\gamma\to -m}\partial/\partial\gamma[(\gamma+m)\Gamma\left(\gamma+\left(\frac{n}{2}\right)\right) = \lim_{\gamma\to -m}\partial/\partial\gamma[(\gamma+m)\Gamma(\gamma)\frac{(-1)^{\frac{n}{2}}\Gamma(1-\gamma)}{\Gamma\left(-\gamma-\frac{n}{2}+1\right)} =$$

$$= \lim_{\gamma\to -m}\{(\gamma+m)\Gamma(\gamma)[\frac{\Gamma\left(-\gamma-\frac{n}{2}+1\right)\frac{\partial}{\partial\gamma}[(-1)^{\frac{n}{2}}\Gamma(1-\gamma)] - (-1)^{\frac{n}{2}}\Gamma(1-\gamma)\frac{\partial}{\partial\gamma}\Gamma\left(-\gamma-\frac{n}{2}+1\right)}{(\Gamma\left(-\gamma-\frac{n}{2}+1\right))^2}]+$$

$$+\frac{(-1)^{\frac{n}{2}}\Gamma(1-\gamma)}{\Gamma\left(-\gamma-\frac{n}{2}+1\right)}\partial/\partial\gamma[(\gamma+\mathrm{m})\Gamma(\gamma)\}=$$

$$=Res_{\gamma=-\mathrm{m}}\Gamma(\gamma).\left[\frac{\Gamma\left(m-\frac{n}{2}+1\right)((-1)^{\frac{n}{2}}.\Gamma'(1+m)(-1)-(-1)^{\frac{n}{2}}\Gamma(1+m)(-1)\Gamma'(m-\frac{n}{2}+1)}{(\Gamma\left(m-\frac{n}{2}+1\right))^2}\right]+$$

$$+\frac{(-1)^{\frac{n}{2}}\Gamma(1+m)}{\Gamma\left(m-\frac{n}{2}+1\right)}\ \mathrm{P.f}\{\Gamma(\gamma)\}_{\gamma=-\mathrm{m}}==(-1)(-1)^m\psi(1+\mathrm{m})\frac{(-1)^{\frac{n}{2}}}{\Gamma\left(m-\frac{n}{2}+1\right)}-\frac{(-1)^m}{m!}\left[\frac{(-1)^{\frac{n}{2}}\Gamma(1+m)(-1)\Gamma'(m-\frac{n}{2}+1)}{(\Gamma\left(m-\frac{n}{2}+1\right))^2}\right]+$$

$$\frac{(-1)^{\frac{n}{2}}\Gamma(1+m)}{\Gamma\left(m-\frac{n}{2}+1\right)}\frac{(-1)^m}{m!}\ \psi(1+\mathrm{m})=(-1)^m\frac{(-1)^{\frac{n}{2}}}{\Gamma\left(m-\frac{n}{2}+1\right)}\left[\ \psi(1+\mathrm{m})+\ \psi\left(m-\frac{n}{2}+1\right)-\psi(1+\mathrm{m})=\right.$$

$$=(-1)^m\frac{(-1)^{\frac{n}{2}}}{\Gamma\left(m-\frac{n}{2}+1\right)}\psi\left(m-\frac{n}{2}+1\right) \qquad (47)$$

where $\psi(z)$ is defined by (43). From(45) and using (47) we have

$$Q^{\lambda-\frac{n}{2}}\,|_{\lambda=\mathrm{m}-\frac{n}{2}}=Q(y)^{-m}==\lim_{\gamma\to-\mathrm{m}}\partial/(\partial\gamma))[(\gamma+\mathrm{m})Q(y)^{\gamma}]=$$

$$=\frac{1}{(-1)^{\frac{v}{2}}\Gamma(m)\pi^{\frac{n}{2}}}\{\frac{(-1)^m}{m!}\psi(1+\mathrm{m})2^{-2m}\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}+\frac{(-1)^m}{m!})2^{-2m}2\ln2\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}+$$

$$+\frac{(-1)^m}{m!}2^{-2m}(-1)\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\ln P_+\right\}--\frac{\frac{(-1)^m}{m!}2^{-2m}}{(-1)^{\frac{v}{2}}\Gamma(m)\pi^{\frac{n}{2}}}\ \mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}(-1)\frac{\Gamma'(m)}{\Gamma(m)\Gamma(m)}=$$

$$\frac{(-1)^{\mathrm{m}-\frac{n}{2}}}{(\mathrm{m}-\frac{n}{2})!}\frac{2^{-2m}}{(-1)^{\frac{v}{2}}\Gamma(m)\pi^{\frac{n}{2}}}\{\psi\left(m-\frac{n}{2}+1\right)\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\ln P_+\right\}+$$

$$+2\ln2\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}-\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\ln P_+\right\}+\ \psi(m)\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}\}=$$

$$=\frac{(-1)^{\mathrm{m}-\frac{n}{2}}}{(\mathrm{m}-\frac{n}{2})!}\frac{2^{-2m}}{(-1)^{\frac{v}{2}}\Gamma(m)\pi^{\frac{n}{2}}}\{\left[\psi\left(m-\frac{n}{2}+1\right)+2ln2+\psi(m)\right]\mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\right\}+\ \mathrm{F}\left\{P_+^{\mathrm{m}-\frac{n}{2}}\ln P_+\right\}\} \qquad (48)$$

where $\psi(z)$ is defined by (43). From (24) and considering (48), we obtain that

$$\mathrm{E}=\mathrm{F}^{-1}\{(-1)^{-m}Q(y)^{-m}\}=\frac{(-1)^{\mathrm{m}-\frac{n}{2}}}{(\mathrm{m}-\frac{n}{2})!}\frac{2^{-2m}}{(-1)^{\frac{v}{2}}\Gamma(m)\pi^{\frac{n}{2}}}P_+^{\mathrm{m}-\frac{n}{2}}+$$

$$+\left[\psi\left(m-\frac{n}{2}+1\right)+2\ln2+\psi(\mathrm{m})\right]P_+^{\mathrm{m}-\frac{n}{2}}-P_+^{\mathrm{m}-\frac{n}{2}}\ln P_+ \qquad (49)$$

is elementary solution of the equation (18) for n even (μ and ν are both even) and m≥(n/2), where

$$L_P^m = \left(\sum_{i=1}^{\mu} \frac{\partial^2}{\partial x_i^2}\right)^m - \left(\sum_{i=\mu+1}^{\mu+\upsilon} \frac{\partial^2}{\partial x_i^2}\right)^m \quad (50)$$

2.2.3.:n even, m≥(n/2), μ and ν odd. In this case m=(n/2),(n/2)+1,(n/2)+2,... , $Q^{-\lambda-\frac{n}{2}}$ has simple poles of order two at λ=m-(n/2) and P_+^{λ} is regular at λ=m-(n/2). Therefore $Q^{-\mathrm{m}}$ is the finite part of $Q^{-\lambda-\frac{n}{2}}$ at λ=m-(n/2), i.e.

$$Q^{\lambda-\frac{n}{2}}\Big|_{\lambda=\mathrm{m}-\frac{n}{2}} = (Q(y)^{-m} = \mathrm{P.f}\{Q^{\gamma}\}_{\gamma=-\mathrm{m}} =$$

$$= \frac{1}{2}\lim_{\gamma\to-\mathrm{m}} \frac{\partial^2}{\partial\gamma}(\gamma+\mathrm{m})^2 (Q(y)^{\gamma}. \quad (51)$$

Using (22), for m≥(n/2),we have

$$\left(\frac{\partial}{\partial\gamma}\right)[(\gamma+m)^2(Q(y)^{\gamma}] = \frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}} \frac{\partial}{\partial\gamma}\left\{\frac{((\gamma+m)^2 F\{P_+^{-\gamma-(n/2))}\}}{.\ 2^{-2\gamma}(\Gamma(-\gamma)\Gamma(1-\gamma-(n/2))cos(\gamma+(n/2))\pi)}\right\} =$$

$$\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}}\{(\gamma+m)^2 . \frac{\partial}{\partial\gamma}\left[\frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right] +$$

$$+ \frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)} . [2(\gamma+\mathrm{m})]\ \}$$

and

$$(1/2)((\partial^2)/(\partial\gamma^2))[(\gamma+m)^2(Q(y)^{\gamma}] =$$

$$\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}}\{(\gamma+m)^2 . \frac{\partial^2}{\partial\gamma^2}\left[\frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right] +$$

$$+\frac{\partial}{\partial\gamma}\left[\frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right](2(\gamma+m)+$$

$$+2.\left[\frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right] +$$

$$(2(\gamma+m)\left[\frac{F\{P_+^{-\gamma-(n/2))}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right]\ \}, \quad (52)$$

therefore

$$\frac{1}{2}\lim_{\gamma\to-\mathrm{m}} \frac{\partial^2}{\partial\gamma^2}(\gamma+\mathrm{m})^2(Q(y)^{\gamma} = \frac{1}{2}\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}}.$$

$$\lim_{\gamma\to -m}\left[\frac{F\{P_+^{-\gamma-(n/2))}\}}{.\ 2^{-2\gamma}\left(\Gamma(-\gamma)\Gamma\left(1-\gamma-\left(\frac{n}{2}\right)\right)\cos\left(\gamma+\left(\frac{n}{2}\right)\right)\pi\right)}\right]2=$$

$$=\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}}.\left[\frac{F\{P_+^{m-(n/2))}\}}{.\ 2^{2m}\left(\Gamma(m)\Gamma\left(1+m-\left(\frac{n}{2}\right)\right)(-1)^{m-\frac{n}{2}}\right)}\right] \quad (53)$$

From (24) and considering (51),we obtain that

$$E=\mathrm{F}^{-1}\{-1)^{-m}(Q(y))^{-m}=\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}}.\left[\frac{\{P_+^{m-(n/2))}\}}{.\ 2^{2m}\left(\Gamma(m)\Gamma\left(1+m-\left(\frac{n}{2}\right)\right)(-1)^{m-\frac{n}{2}}\right)}\right] \quad (54)$$

is elementary solution of the equation (18) for n even(μ and ν are both odd) and m≥(n/2).

2.2.4.:n even, m<(n/2), μ and ν odd. We know from ((Gelfand and Shilov, 1964),page 352), that if μ and ν are both odd, the generalized function P_+^{λ} has simple poles at λ=-1,-2,...-((n/2)-1) and poles of second order at λ=-(n/2)-s,s=0,1,2,..

Now taking into account that m<(n/2),then m≠(n/2)+s, s=0,1,2,... and the distributions $Q^{-\lambda-\frac{n}{2}}$ and P_+^{λ} have simple poles at λ=m-(n/2).

On the other hand using the formula

$$\Gamma(z)\Gamma(1-z)=\frac{\pi}{\sin(z\pi)} \quad (55)$$

the formula (22) can be rewritten in the following form

$$(Q(y)^{-\lambda-\frac{n}{2}}=a_{\upsilon,\pi}.\frac{\sin(\lambda\pi))\Gamma(-\lambda)}{2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)}\mathrm{F}\{P_+^{\lambda}\} \quad (56)$$

where

$$a_{\upsilon,\pi}=\frac{1}{(-1)(-1)^{\frac{\upsilon-1}{2}}\pi^{\frac{n}{2}-1}} \quad (57)$$

Now taking into account the definition of finite part of $(Q(y)^{-\lambda-\frac{n}{2}}$ at λ=m-(n/2) for m<(n/2), we have

$$Q^{\lambda-\frac{n}{2}}\,|_{\lambda=m-\frac{n}{2}}=(Q(y)^{-m}=\mathrm{P.f}\{Q^{-\lambda-\frac{n}{2}}\}_{\lambda=m-\frac{n}{2}}=\lim_{\lambda\to-(\frac{n}{2}-m)}\frac{\partial}{\partial\gamma}(\lambda+\frac{n}{2}-\mathrm{m})(Q(y)^{\gamma}=$$

$$=a_{\upsilon,\pi}\lim_{\lambda\to-(\frac{n}{2}-m)}\frac{\partial}{\partial\gamma}\left\{\frac{\sin(\lambda\pi))\Gamma(-\lambda)}{2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)}(\lambda+\frac{n}{2}-\mathrm{m})\mathrm{F}\{P_+^{\lambda}\}\right\}$$

$$=a_{\upsilon,\pi}\lim_{\lambda\to-(\frac{n}{2}-m)}\left\{\frac{2^{2\lambda+n}(\cos(\lambda\pi)\Gamma\left(\lambda+\left(\frac{n}{2}\right)\right).\frac{\partial}{\partial\gamma}[\sin(\lambda\pi))\Gamma(-\lambda)\left(\lambda+\frac{n}{2}-\mathrm{m}\right)\mathrm{F}\{P_+^{\lambda}\}]}{[2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)]^2}\right. -$$

$$\left.-\frac{[\sin(\lambda\pi))\Gamma(-\lambda)\left(\lambda+\frac{n}{2}-\mathrm{m}\right)\mathrm{F}\{P_+^{\lambda}\}].\frac{\partial}{\partial\gamma}[2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)]}{[2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)]^2}\right\}=$$

$$a_{\upsilon,\pi}\lim_{\lambda\to-(\frac{n}{2}-m)}\left\{\frac{.\pi\cos(\lambda\pi)\Gamma(-\lambda)\left(\lambda+\frac{n}{2}-\mathrm{m}\right)\mathrm{F}\{P_+^{\lambda}\}]-[\sin(\lambda\pi))(-1)\Gamma\prime(-\lambda)\left(\lambda+\frac{n}{2}-\mathrm{m}\right)\mathrm{F}\{P_+^{\lambda}\}}{2^{2\lambda+n}(\cos(\lambda\pi)\Gamma\left(\lambda+\left(\frac{n}{2}\right)\right)}\right.+$$

$$+\frac{[\sin(\lambda\pi))\Gamma(-\lambda)\frac{\partial}{\partial\gamma}\left(\lambda+\frac{n}{2}-m\right)F\{P_+^{\lambda}\}}{2^{2\lambda+n}(\cos(\lambda\pi)\Gamma\left(\lambda+\left(\frac{n}{2}\right)\right)} - \frac{[\sin(\lambda\pi))\Gamma(-\lambda)\left(\lambda+\frac{n}{2}-m\right)F\{P_+^{\lambda}\}].}{[2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)]^2} -$$

$$-[\frac{2^{2\lambda+n}2\ln 2(\cos(\lambda\pi)\Gamma(\lambda+\left(\frac{n}{2}\right)-\pi 2^{2\lambda+n}(\sin(\lambda\pi)\Gamma(\lambda+\left(\frac{n}{2}\right))+2^{2\lambda+n}(\cos(\lambda\pi)\Gamma'(\lambda+\left(\frac{n}{2}\right))}{[2^{2\lambda+n}(\cos(\lambda\pi)\Gamma(\lambda+(n/2)]^2}\} \quad (58)$$

Now using that

$$\lim_{\lambda\to-(\frac{n}{2}-m)} \left(\lambda+\frac{n}{2}-m\right)\{P_+^{\lambda}\} = Res\{P_+^{\lambda}\}_{\lambda=(\frac{n}{2}-m)} = \frac{(-1)^{\frac{n}{2}-m-1}}{\left(\frac{n}{2}-m-1\right)!}\delta^{\left(\frac{n}{2}-m-1\right)}(P) \quad (59)$$

$$\lim_{\lambda\to-(\frac{n}{2}-m)} \frac{\partial}{\partial\gamma}\left(\lambda+\frac{n}{2}-m\right)\{P_+^{\lambda}\} = \mathrm{P.f}\{P_+^{\lambda}\}_{\lambda=m-\frac{n}{2}} = P_+^{m-\frac{n}{2}} \quad (60)$$

and $\lim_{\lambda\to-(\frac{n}{2}-m)} \sin(-\lambda\pi) = 0$ (61)

$$\lim_{\lambda-(\frac{n}{2}-m)} \cos(-\lambda\pi) = \cos((n/2)-m)\pi) = (-1)^m(-1)^{\frac{n}{2}} \quad (62)$$

$$\lim_{\lambda\to-\left(\frac{n}{2}-m\right)} \Gamma(-\lambda) = \Gamma\left(\frac{n}{2}-m\right), \frac{n}{2}\geq m, \quad (63)$$

We have

$$Q(y)^{-m} = \mathrm{P.f}\{Q^{\lambda-\frac{n}{2}}\}_{\lambda=-(m-\frac{n}{2})} = a_{\upsilon,\pi}\frac{(-1)^{\frac{n}{2}-m-1}(-\pi)2^{-2m}}{\Gamma(m)}\delta^{\left(\frac{n}{2}-m-1\right)}(P) \quad (64)$$

From(56)and considering (64),we obtain that

$$E=F^{-1}\{(-1)^{-m}Q(y)^{-m}\} = \frac{1}{(-1)^{\frac{\mu-1}{2}}\pi^{\frac{n}{2}-1}}\frac{1}{\Gamma(m)2^{2m}}.\delta^{\left(\frac{n}{2}-m-1\right)}(P) \quad (65)$$

is elementary solution of the equation (18) for n even(μ and ν are both odd) and m<(n/2).

5. CONCLUSION

It´s clear from (24) that if $u(x_1, \ldots x_n)= E(x_1, \ldots x_n)$ is elemental solution of the L_P^m then the solution of the equation (18) is given by the following formula

$$u = u(x_1, \ldots x_n)=f(x_1, \ldots x_n)*E, \quad (66)$$

for $f(x_1, \ldots x_n)$ generalized function with compact support ,where the symbol $*$ we means convolution and E is defined in the following form:

$$E(x_1, \ldots x_n)=A_{m,n}=P_+^{m-\frac{n}{2}} \text{ if n is odd}, m < \left(\frac{n}{2}\right) \text{ and } m > (n/2), \quad (67)$$

$E(x_1,\ldots x_n)$=$B_{m,n}P_+^{m-\frac{n}{2}}+C_{m,n}\delta^{\left(\frac{n}{2}-m-1\right)}(P)$if n is even, $m<\left(\frac{n}{2}\right)$, μ and ν are both even, (68)

$E(x_1,\ldots x_n)$=$D_{m,n}P_+^{m-\frac{n}{2}}-F_{m,n}P_+^{m-\frac{n}{2}}lnP_+ if\ n\ is\ even, m\geq(n/2), \mu\ and\ \nu\ are\ both\ even,$ (69)

$E(x_1,\ldots x_n)$=$G_{m,n}P_+^{m-\frac{n}{2}}$if n is even, m ≥ (n/2), μ and ν are both odd (70)

and

$E(x_1,\ldots x_n)$=$H_{m,n}(P)\delta^{\left(\frac{n}{2}-m-1\right)}$if n is even, m < (n/2), μ and ν are both odd. (71)

where

$A_{m,n}$=$\frac{(-1)^m\Gamma((n/2)-m))}{2^{2m}\Gamma(m))\,(-1)^{\frac{v}{2}}\quad\pi^{\frac{n}{2}}}$, $n\ odd, m<\left(\frac{n}{2}\right)$ and $m>\left(\frac{n}{2}\right)$, (72)

$B_{m,n}=\frac{(-1)^m\Gamma((n/2)-m))}{2^{2m}\Gamma(m))\,(-1)^{\frac{v}{2}}\quad\pi^{\frac{n}{2}}}$, n even, $m<\left(\frac{n}{2}\right)$, μ and ν are both even, (73)

$C_{m,n}=\frac{(-1)^m}{(-1)^{\frac{n}{2}}\ \ 2^{2m}\Gamma(m))\,(-1)^{\frac{v}{2}}\quad\pi^{\frac{n}{2}}}\left[\psi\left(\frac{n}{2}-m\right)+2\ln2+\psi(m)\right]$if
n even, $m<\left(\frac{n}{2}\right)$, μ and ν are both even, (74)

$D_{m,n}=\frac{(-1)^{\frac{n}{2}}}{\left(m-\frac{n}{2}\right)!2^{2m}\Gamma(m))\,(-1)^{\frac{v}{2}}\quad\pi^{\frac{n}{2}}}.\left[\psi\left(m-\frac{n}{2}+1\right)+2\ln2+\psi(m)\right]$

$if\ n\ is\ even, m\geq(n/2), \mu\ and\ \nu\ are\ both\ even,$ (75)

$F_{m,n}=\frac{(-1)^{\frac{n}{2}}}{\left(m-\frac{n}{2}\right)!2^{2m}\Gamma(m))\,(-1)^{\frac{v}{2}}\quad\pi^{\frac{n}{2}}}\ \ if\ n\ is\ even, m\geq(n/2), \mu\ and\ \nu\ are\ both\ even,$ (76)

$G_{m,n}=\frac{1}{(-1)2^{2m}\Gamma(m)\Gamma(1+m+(n/2))\,(-1)^{\frac{v-1}{2}}\ \ (-1)^{m-\frac{n}{2}}\ \ \pi^{\frac{n}{2}-1}}$

if n is even, m ≥ (n/2), μ and ν are both odd (77)

and

$H_{m,n}=\frac{1}{2^{2m}\Gamma(m))\,(-1)^{\frac{\mu-1}{2}}\quad\pi^{\frac{n}{2}-1}}$if n is even, m < (n/2), μ and ν are both odd.

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